1. ‘Classify’ the following differential equations, i.e., determine whether the following equations are homogeneous/non-homogeneous, linear/non-linear, 1st order/2nd order/…, ordinary differential equations:
   a) \( t \frac{dy}{dt} + y \sin t = 0; \)
   b) \((1 + t^2) \frac{dy}{dt} + 2ty + 3 = 0; \)
   c) \(\frac{dy}{dt} + \frac{d^2y}{dt^2} = 3t; \)
   d) \(4 \frac{d^2y}{dt^2} + \frac{dy}{dt} + 6y = e^t + \sin 2t; \)
   e) \(u \frac{d^4u}{dt^4} + \left(\frac{du}{dt}\right)^2 = 0. \)

2. Find the general solution of the following differential equations:
   a) \(\frac{dy}{dt} + y \sin t = 0; \)
   b) \((1 + t^2) \frac{dy}{dt} + ty = (1 + t^2)^{3/2} \)

3. Solve the following initial/boundary value problems, and express your answers in the form “y=y(t)”.
   a) \(\frac{dy}{dt} + 4ty = 2te^{2t^2}, \quad y(1) = 5. \)
   b) \(y \frac{dy}{dt} - 2ty^4 = 0, \quad y(0) = \frac{1}{2}. \)

4. Find the general solution of the following differential equations: \(\frac{dy}{dt} = \frac{y}{t} - \frac{y^3}{t^3}. \)

5. Find the general solution of the following differential equations:
   a) \(\frac{d^2y}{dt^2} - 7 \frac{dy}{dt} + 10y = 0; \)
   b) \(\frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 9y = 0; \)
   c) \(\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 3y = 0. \)

6. Solve the following initial value problem:
   \(\frac{d^2y}{dt^2} - 3 \frac{dy}{dt} - 4y = 0; \quad y(0) = 4, \quad y'(0) = 0. \)

7. Use the method of undetermined coefficients to find a particular solution of the following differential equations:
   \(y'' + 3y = 6t^3 - 6. \)

8. Find the general solution of the following differential equations:
i) \( y'' - 3y' + 2y = 3e^{2t} + e^{3t} \).
ii) \( y'' + 4y = \cos 2t \).

9. Solve the following initial value problem:
\[ y'' + 4y' + 3y = 2e^{-3t}; \quad y(0) = 0, \quad y'(0) = 3. \]

10. A tank containing \( S_0 \) lb of salt dissolved in 200 gallons of water. Starting at time \( t = 0 \), water containing \( \frac{1}{2} \) lb of salt per gallon enters the tank at rate of 4 gal/min, and well stirred solution leaves the tank at the same rate. Find the concentration of salt in the tank at any time \( t > 0 \).

Some suggested answers:
1 a) homogeneous linear 1st order ODE
1 b) non-homogeneous linear 1st ODE
1 c) non-homogeneous non-linear 3rd order ODE
1 d) non-homogeneous linear 2nd order ODE
1 e) homogeneous non-linear 4-th order ODE.
2 a) \( y = Ce^{\cos t} \)
2 b) \( y = \frac{1}{\sqrt{1+t^2}} \left( t + \frac{t^3}{3} + C \right) \)
3 a) \( y = e^{-2t^2} (t^2 + 5e^2 - 1) \)
3 b) \( y = \frac{1}{\sqrt{8 - 3 \ln(1+t^2)}} \)
4 y = \( \frac{t}{\sqrt{2(\ln |t| + C)}} \)
5 a) \( y(t) = Ae^{2t} + Be^{3t} \)
5 b) \( y(t) = (At + B)e^{-3t} \)
5 c) \( y(t) = Ae^{-t} \cos \sqrt{2}t + Be^{-t} \sin \sqrt{2}t \)
6. \( y(t) = \frac{4}{5}e^{4t} + \frac{16}{5}e^{-t} \)
7. \( y_p(t) = 2t^3 - 4t - 2 \)
8 a) \( y(t) = Ae^{t} + Be^{2t} + 3te^{2t} + \frac{1}{2}e^{3t} \)
8 b) \( y(t) = A \cos 2t + B \sin 2t + \frac{t}{4} \sin 2t \)
9. \( y(t) = 2e^{-t} - 2e^{-3t} - te^{-3t} \)
10. the concentration \( c(t) \) of salt in the tank is given by
\[ c(t) = \frac{S_0}{200} e^{-0.02t} + \frac{1}{2} (1 - e^{-0.02t}). \]