ANALYSIS OF KALAH

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ABSTRACT

The aim is to suggest possible winning strategy for the first player and possible counter attacks for the second player for two cases of Kalah. The two cases of the game that will be analysed are Kalah (1, m) and Kalah (n, 1). Kalah (1, m) is the case where there is only one pit and varying number of seeds in each pit. Kalah (n, 1) is the case where one seed and varying number of pits. Other than suggesting possible winning strategy and counter attacks, any observations with and without its proof and/or trends of the two cases was noted down as well.

INTRODUCTION

The game usually involves two players (North and South) and a board that is placed between the two players. The two larger pits on the extreme right and extreme left of the board known as the Kalaha. They are left empty at the beginning of the game. This is basically a sowing game and two cases of Kalah are included in this research. Firstly, it is the case of Kalah (1, m) in which there will only be one pit on each side of the board for each respective player each bearing m number of seeds. Secondly, the case of Kalah (n, 1) that has n pits on each side of the board each bearing one seed.

Rules of the Game

The first player can scoop up all the pieces to start his move from any non-empty pit on his side of the board and sows in anti-clockwise direction, starting with the next pit. He cannot start his move using the pieces on the opponent’s side of the board. The player's Kalaha, the player’s pits and the opponent’s pits, excluding the opponent’s Kalaha, are included in the sowing.

For normal play case, if the last seed lands in the player’s Kalaha, his score is increased by 1 and he retain the right to continue playing. As for alternate play, his score also increase by one but the player loses his turn after every move that he has made even if the move is the one in which his last seed lands in his Kalaha. The player gets to capture his seed if his last seed lands in an empty pit on his side of the board and the opponent’s pit, which is opposite to the empty pit of his, is also empty. He gets to capture more than one seed if the opponent’s pit opposite to the empty pit in which the last seed of the player lands is non-empty. The player loses his turn without increasing his score when his last seed lands in a non-empty pit that can either belongs to him or his opponent.

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The seeds that are being captured or the seeds that has entered both players’ Kalaha do not re-enter the game. The game value of this game thus depends on the configuration of the active seeds or seeds that are not captured. If the player is left with no seeds on his side, the game is over and the remaining seeds are added to the opponent’s score pit.

Possible Outcomes of a Move

First of all, Home or Kalah-move happens if the last seed lands in his score pit. Secondly, move to capture is a move in which the last seed lands in an empty pit on the player’s side of the board, it is added to his total and any seeds in the opposite pit are added as well. Thirdly, null capture takes place if the last seed played ends in an empty pit on the player’s side and the opponent's pit that is directly opposite is also empty. For this case, the last seed is still placed in the player’s Kalaha.

Observations

Firstly, the game value of this game decreases as with each turn that any of the two players get to sow. Secondly, the player’s score increases by at least 1 for every capture he makes. Thirdly, if in the case for normal play, player 1 starts by moving the seed that will end up in his home, he will lead by at least one capture. Thus, he can optimise his chances of winning by leading by at least one capture. Fourthly, player can move to capture, move to defend, move to go home, make a random move or mirror the moves of the opponent.

Objective of the Game

The aim is to be the player with the most number of seeds in his Kalaha.

KALAH (1,m)

This is the case with one pit and m number of seeds where m is any positive integer. There are two forms for this part of Kalah - Normal play and alternate play.

Strategy for Player 1

For both forms, there exists no possible strategy for player 1. Player 1 can only sow his seeds and lose his turn when his last seed did not end up in his Kalaha for the case of normal play or when he has made a move for the case of alternate play. Player 1 will regain his turn when player 2 loses turn. This will continue until one of the players does not have any more counter in his pits. The remaining seeds will then go into the opponent’s Kalaha. The player with the larger number of seeds is the winner. If both players have the same number of seeds, it is a draw.

Strategy for Player 2 (Counter Attacks)

For both normal and alternate play of Kalah, there is no counter attack available for P2.

Observations

It can be observed that if a player starts off with 0 (mod 3) number of seeds, his last seed will end his pit. If a player starts off the game with 1 (mod 3) number of seeds, his last seeds will end in his Kalaha. If a player starts off the game with 2 (mod 3) number of seeds, his last
seeds will end up in his opponent’s pit. The result for the game where both players has one seed each is always a draw for both normal play and alternate play.

In the case where the players follow the rule as stated for normal play, for the starting configuration of $111...1$ (ternary representation) or $3^0 + 3^1 + 3^2 + ... + 3^m$ (decimal representation), P1 always loses. The number of turns that the player gets to retain is equivalent to the number of “1”s that the player has.

Still following the rules of normal play and assuming the player started the game with $k$ number of “1”s, the recursive formula for the number of seeds, written in its base 3 or ternary representation, in the player’s Kalaha is of the form:

$$1 + 11 + ... + 111...1 + k$$

where $111...1$ has k-1 number of “1”s and k is any positive integers of base 10.

The recursive formula for the number of seeds in the opponent’s Kalaha will be:

$$1 + 11 + ... + 111...1$$

where $111...1$ has k number of “1”s and k is any positive integers of base 10.

The recursive formula for the difference in the number of seeds between the two players is:

$$11...1 - k$$

where $11...1$ has k number of “1”s and k is any positive integers of base 10.

Last but not least for the case of normal play, the player who start off with $10111...1$ (ternary representation) number of seeds always wins all the seeds that is on the board.

As for the alternate play, excluding the case where $m = 3$, all other configurations will not result in P1 winning all the seeds that are available on the board. The difference between the numbers of seeds in both players’ Kalaha is even and at most 4 or zero for any m excluding the case when $m = 3$.

**KALAH (n, 1)**

This is the case where there is $n$ number of pits on each side of the board where $n$ is any positive integer. There exists only one seed in each of the pits on both sides of the board. For both forms of play, suggested opening moves are given. Other than that, counter examples to explain the failure of a move that is to be taken over the other four moves at any time after the first move has been made is given.

**Normal Play**

The classic starting move, the first two moves that P1 makes, is to move to Kalaha and then to capture whenever it is possible. The strategies for P1 that has been tested and failed are: i. Capture, Kalaha ii. Defend, capture, Kalaha. As for P2, the failed strategies are: i. Mirror ii. Capture, Kalaha iii. Kalaha iv. Defend, capture, Kalaha.

Some of the observations made are firstly, there are configurations when the player can have a series of moves in which his last seed lands in his Kalaha. Secondly, in order for the player to be able to retain his turn if a pit that is $n$ steps away from his Kalaha contain $n$ number of seeds. Lastly, the game is always a draw (when $n$ is odd) or win (when $n$ is even) for the first player if first player always follow the strategy of moving the seeds nearest to his Kalaha and the second player always mirror his moves.
Alternate Play

The strategy for P1 obeying the rules for alternate play will be to move to home then to capture whenever possible. Random move is not encouraged. As for P2, the strategies that have been tested and have shown to fail are: i. mirror, ii. Random, Kalaha, iii. Defend, Kalaha.

DISCUSSION AND CONCLUSION

Kalah has been around for quite some time and has provided fun for those who have played it. Interestingly, Kalah bears the meaning of losing. In brief, it is a challenging game to analyses and it can get very complicated even for the case of Kalah (1, m) and Kalah (n, 1) that is supposed to be the easier cases for Kalah. Thus far, I am only able to come up with observations, possible starting strategies for player one and counter examples to explain the failure of certain strategies. I am sure more can be done in order to solve the Kalah.

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