

Fifty Years of Mathematics in Singapore: A Personal Perspective

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In tandem with the development of Singapore as a nation, mathematics in Singapore underwent a dramatic transformation in the first 50 years of the country's history. What follows is a brief account, based on both my own recollection and inputs from colleagues, of the events that shaped the current state of mathematics in Singapore.

When Singapore became independent on 9 August 1965, it had two mathematics departments, one at the University of Singapore and the other at Nanyang University. The history of the Department of Mathematics at the University of Singapore traces back to 1929 when Raffles College was established (see Peng [5]). The major mathematical figure at the time was Alexander Oppenheim who was a student of G. H. Hardy at Oxford University and received his PhD from the University of Chicago under L. E. Dickson. Oppenheim joined Raffles College in 1931 and was Head of the Department for the next 28 years until 1959, when he was appointed Vice-Chancellor of the University of Malaya which evolved from Raffles College in 1949. In 1965, Oppenheim retired from his position and left Singapore. He was later honored by the government of Malaya with the award of Tan Sri and was knighted by the British monarchy. Apart from his contributions to the development of mathematics in the early days of Singapore, Oppenheim also published extensively in number theory and is best known for formulating the conjecture named after him on the representation of numbers by quadratic forms of several variables. A definitive solution of the conjecture was given by the Fields Medalist Grigori Margulis in 1987.

The Department of Mathematics at Nanyang University was established in 1955 when the university was founded. In the 1960s and 70s, driven by a pioneering spirit, a group of the Department's earlier graduates who returned from the UK and Canada with doctoral degrees succeeded in securing funding from the Lee Foundation to set up the Lee Kong Chian Centre for Mathematical Research. During this period, the Department played a significant role in promoting mathematical activities in Singapore and Southeast Asia. These included supporting overseas visitors to Singapore, publishing the journal *Nanta Mathematica*, taking the lead in forming the Southeast Asian Mathematical Society, establishing exchange and training programs in mathematics with universities in the region, as well as organizing the First Franco-Southeast Asian Mathematical Conference in 1979 (jointly sponsored by the French-Singapore Scientific Cooperation Program). The conference was a major event as it featured ten leading French mathematicians among the list of invited speakers. These included Jean Dieudonné, Jacques-Louis Lions and Fields Medalist Laurent Schwartz.

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² In this article, Chinese names are written with surnames first.



Left to right: J-L. Lions, L. Schwartz and J. Dieudonné³

The departments of mathematics at the two universities differed in research interests. Due to its colonial origin, the Department at the University of Singapore was headed by expatriate faculty from the very beginning, and this continued until 1980 when K. K. Sen retired after a brief one-year headship to return to India. The research areas during the period 1950–1970 were wide and diverse: number theory (Oppenheim), classical algebraic geometry (Dan Pedoe), combinatorics (Eric Milner (especially combinatorial set

theory) and Richard Guy (who also worked in game theory and number theory)), combinatorial group theory (Malcolm Wicks), analysis (U. C. Guha), topology (E. J. Brody), statistics (P. H. Diananda (who also worked in analysis)), applied mathematics (Peter Lancaster and Rex Westbrook⁴), astrophysics (K. K. Sen and S. J. Wilson), history of Chinese mathematics (Lam Lay Yong). Many were publishing high quality scholarly papers. In fact, the first paper in the *Annals of Mathematics* from Singapore (by Brody on lens spaces) appeared in 1960. On the other hand, it was not until 1965 that the Department saw its first Singaporean student return from abroad with a PhD (Peng Tsu Ann, PhD in group theory from Queen Mary College). Peng led the Department as Head from 1982 to 1996 and oversaw the Department's rapid growth during that period. By the early 1970's, the Department had among its junior faculty several alumni who returned from overseas studies. They included Leong Yu Kiang (PhD in group theory, Australian National University), Ng How Ngee (PhD in representation theory, University of Illinois at Urbana-Champaign), Louis Chen (PhD in statistics, Stanford University) and Cheng Kai Nah who took the unusual step for a Singaporean in those days of pursuing a doctorate in continental Europe (PhD in group theory, Johannes Gutenberg-Universität Mainz).

At Nanyang University, research was very much focused on graph theory (Teh Hoon Heng and his students). While the faculty's research interests included areas such as lattice theory (Chen Chuan Chong and Koh Khee Meng), analysis (Lee Peng Yee) and operations research (Chew Kim Lin), graph theory was arguably the most prominent among them. In 1985, the journal *Graphs and Combinatorics* was published by Springer-Verlag with Teh as its founding managing editor.

When Singapore declared independence, the University of Singapore had awarded its first PhD (Lancaster, 1964, under the supervision of Dan Pedoe) and the Singapore Mathematical Society (formerly the Malayan Mathematical Society) was 13 years old. The total number of mathematicians engaging in teaching and research in the country was not more than 20. However, by the time I joined the Department of Mathematics at the University of Singapore in April 1974 as a lecturer, it already had a faculty strength of 17. Research was for personal interest and not a requirement for career advancement. Nevertheless, the pursuit of scholarship for its own sake was very much alive. I remember that together with some junior colleagues, we ran a weekly seminar to read *Cours d'arithmétique* (A Course in Arithmetic) by Jean-Pierre Serre, and a

³ Photo courtesy of Leong Yu Kiang

⁴Richard Guy, Peter Lancaster, Eric Milner and Rex Westbrook later moved successively to the University of Calgary between the 1950s and 60s, and were sometimes jokingly referred to as "the Singapore mafia".

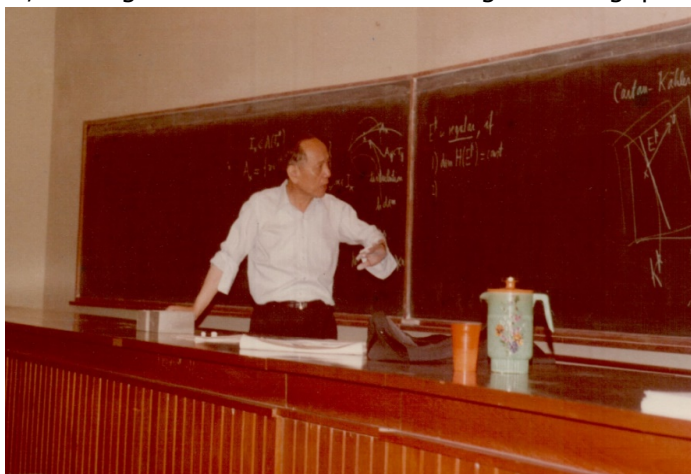
seminar on John Milnor's *Topology from the Differentiable Viewpoint*, even though none of us worked in the related fields.

The Singapore Mathematical Society viewed promoting public interest in mathematics to be a key mission. To this end, it regularly held public lectures delivered by local mathematicians or, on rare occasions, visiting mathematicians. In addition, the Society also played an active role in identifying budding mathematical talents. From the early days, it organized an annual inter-school mathematical competition. Not long after I joined the Department, I was recruited by several junior colleagues (primarily Louis Chen, Leong Yu Kiang, Ng How Ngee) to get involved in the organization of the annual competition. This included liaising with schools, setting competition problems, marking the solutions, getting the prizes for winners, arranging for the engraving of names of winners on the plaque, as well as planning and organizing prize giving ceremonies. Separately, there were two publications under the auspices of the Society: *Mathematical Medley* (founded in 1973) aimed at the general public, school teachers and students, and the *Bulletin of the Singapore Mathematical Society* (formerly *Bulletin of the Malayan Mathematical Society*, founded in 1955) which was devoted to mathematics at the research level. The latter has since been discontinued as attention and interest of local mathematicians shifted to publishing their research papers solely in international journals.

Public research funding for mathematics, and basic science in general, was not available before 1980. Despite this, mathematical research in Singapore was not impeded. In a compendium on research publications between 1949 and 1980 at the University of Singapore printed in the *Straits Times*, no less than 320 papers were published during this period, of which 174 appeared in international refereed journals. On the other hand, attending a conference outside Singapore was very rare and required the use of personal funds, support from a private foundation or an international funding agency. My very first participation in an international conference was at the 1978 International Congress of Mathematicians (ICM) in Helsinki and was funded by the International Mathematical Union. The first time I attended an international meeting in my area of research was six years after joining the Department, on my way for sabbatical leave in the US, making a stopover in Europe for the conference. It was also the first time that I met members of the research community with whom I had corresponded for a number of years. (As an aside, to give a sense of the limited facilities available at the time, my office in the Department during the first six years had no telephone. To make or receive phone calls, I had to use the one on the department secretary's desk on another floor of the building. A buzzer in my office would sound if the secretary received a phone call for me.)

Despite the difficulties, with some effort it was possible to secure funding from private sources, such as the Lee Foundation, to organize small-scale meetings in Singapore.

This enabled, for example, the visit of Chern Shiing-Shen, one of 20th century's great geometers, to Singapore in June 1980. When I met him at the Helsinki Congress and extended an invitation to him to visit Singapore, an English translation of his autobiographical article 学算四十年 (*Forty years of mathematical studies*) had appeared in the *Mathematical Medley* [3] (my colleague Leong Yu Kiang and I were writing a series of articles called "Notes on Mathematicians" for the *Medley*).



Chern Shiing Shen, Singapore, June 1980³

The translated article was one of them). Chern accepted the invitation and spent a week in Singapore. More than 400 people attended Chern's public lecture. The auditorium was packed and late comers had only standing room. This was an unprecedented success for a public mathematical lecture in Singapore (ten years later the Fields Medalist Michael Atiyah gave a public lecture to an audience of almost 700, a number which has not been surpassed to date for such lectures).

Research funding for the basic sciences began in 1980 upon the merger of Nanyang University and the University of Singapore to form the National University of Singapore (NUS). The 1980s was a period of rapid growth in mathematical activities. With the merger, the Lee Kong Chian Centre for Mathematical Research was then a centre in the new department in NUS. It provided financial support for mathematical conferences organized by the Department and the Society. An international conference was held almost every year during this period, each devoted to a specific field. These included group theory, analysis and topology. Each conference brought in leading figures in the field.



Singapore Group Theory Conference, June 1987⁵

Particularly worth noting is the Singapore Group Theory Conference held in 1987 (see [2]) where the invited speakers included Walter Feit, John Thompson, Graham Higman,

⁵ Photo courtesy of the Department of Mathematics, National University of Singapore

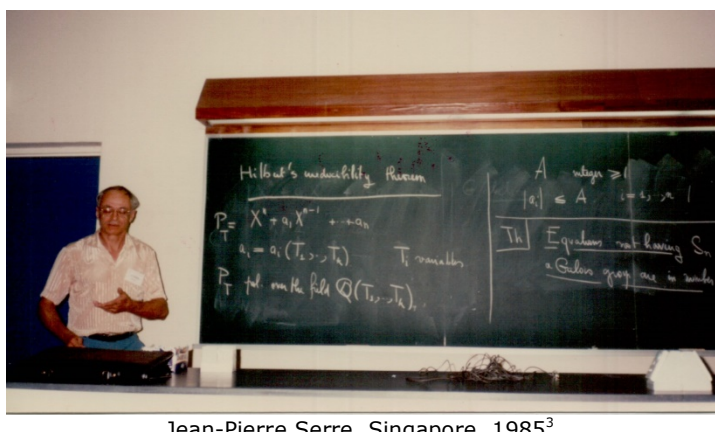


Back row: Tan Sie Keng, Leong Yu Kiang, Brian Hartley, Karl Gruenberg, Michio Suzuki, Michel Broué, Derek Robinson, Jean-Pierre Serre, John Cannon, Malcolm Wicks
Front row: Peng Tsu Ann, Cheng Kai Nah, Walter Feit, Bernhard Neumann, Graham Higman, S.I. Adian, John Thompson, A.I. Kostrikin, Noboru Ito

Singapore Group Theory Conference, June 1987⁶

Michio Suzuki and Jean-Pierre Serre. Between 1980 and 1990, the Centre supported many short-term overseas visitors, most prominent of whom were Fields Medalists René Thom, Michael Atiyah, Jean-Pierre Serre and John Thompson. Serre, in particular, first visited Singapore in February 1985 (see [4]). He made several return visits, most recently in 2009.

The second 15 years (1980-1995) of Singapore's independence also saw the Department grow in faculty strength. At one point it exceeded 106. Most major fields of research were represented: logic, combinatorics, algebra (especially group theory), number theory, representation theory, analysis, differential geometry, algebraic and differential topology, differential equations, mathematical physics, computational mathematics, operations research, probability and statistics.



Jean-Pierre Serre, Singapore, 1985³

⁶ Photo courtesy of Peng Tsu Ann

This was also a period during which talented NUS graduates pursued PhD studies at some of the best departments in the US, funded either by the University or by the host institutions. However, not all of them stayed on in academia. In 2015, ten were on the faculty at the NUS: Tay Yong Chiang (PhD in computer science, Harvard University), Tan Eng Chye (now Provost of NUS) and Lee Soo Teck (PhD in representation theory, Yale University), Chan Heng Huat (PhD in number theory, University of Illinois at Urbana-Champaign), Victor Tan (PhD in number theory, UCLA), Toh Kim Chuan (PhD in operations research, Cornell University), Loke Hung Yean (PhD in representation theory, Harvard University), Loh Wei Liem and Chan Hock Peng (PhD in statistics, Stanford University), and Teo Chung Piaw (PhD in operations research, MIT). The Department also made a concerted effort to recruit promising young mathematicians, some of whom are today acknowledged leaders in their fields. In recent years, the search for global talents, at both the junior and senior levels, has intensified with notable success.

In 1998, the University set up the Department of Statistics and Applied Probability, and the group in statistics moved to the new department. This move resulted in the reduction of faculty size in the Department of Mathematics, and the number of tenure track faculty since then hovers between 55 and 65. Despite this, the quality of mathematical research continued in the upward trend. Evaluation of research performance shifted from the number of papers published in international refereed journals to the quality and significance of work done.



Institute for Mathematical Sciences⁷

Mathematical research in Singapore was marked with a new milestone in 2000 with the establishment of the Institute for Mathematical Sciences (IMS) at the National University of Singapore. The idea of a mathematical institute in Singapore dates back to the 1980s following the enormous success of the MSRI—Mathematical Sciences Research Institute—in Berkeley. (I recall at the party given by Chern Shiing-Shen at his residence (with a spectacular view of the San Francisco Bay) during the 1986 ICM in Berkeley,

⁷ Photo courtesy of Louis Chen

Chern showed Peng Tsu Ann and me his “Director’s chair”, a gift from the Institute which he served as its Founding Director from 1981 to 1984. The idea of a mathematical institute in Singapore modeled after the MSRI later became a regular topic of conversation between Peng and several of us in the Department). Proposals for such an institute were submitted to the University during the interim years. With the country’s decision to move into knowledge economy in the new century, the Ministry of Education gave approval to provide initial funding for such an institute to be set up within the NUS. The IMS organizes programs and workshops with cooperation from the Department of Mathematics. Louis Chen was appointed the Founding Director and served until 2013. Roger Howe of Yale University was the Chair of the IMS Scientific Advisory Board during the period 2001–2010 and was succeeded by Siu Yum Tong of Harvard University. Occupying two double-story colonial houses in Prince George’s Park of the hilly Kent Ridge campus, IMS hosts more than 700 mathematical scientists every year from Singapore and around the world, who give lectures or participate in research collaboration at the Institute. Together the visitors generate a high level of vigor, vibrancy and scientific activities not previously seen in Singapore.

In 2005, Nanyang Technological University established the School of Physical and Mathematical Sciences with Ling San as its Head. The Division of Mathematics now numbers more than 20 on its tenure track faculty and covers research areas in coding theory, logic, algorithms and theoretical computer science, computational mathematics, number theory, probability and topology. Within a short span of ten years, NTU has established a research-intensive division in mathematics. A number of its research groups, for example the group in coding theory, are highly regarded internationally.

Along with increased research funding, the Ministry of Education also provided more scholarships for graduate education. This led to a significant growth of the doctoral program in the mathematical sciences. By 2005 the number of PhD students in the Department had exceeded 60. That number went past 100 by 2015, counting in all the students enrolled in doctoral mathematical programs in Singapore. Many graduates have successfully secured postdoctoral positions at leading research universities in Europe, US and Canada (for example, Oxford, Vienna, Münster, Stanford, Purdue, Toronto).

The Department of Mathematics at NUS moved to its newly renovated building in two



phases, beginning in late 2009 and completing more than a year later. The new home was designed with the objective of creating an environment conducive to teaching, learning and research. For the first time since expanding its doctoral program, faculty members and graduate students have offices in the same building which also houses seminar rooms and a lounge—a working environment that is a far cry from that which I experienced many years before.

Mathematics Lounge, NUS⁵



Mathematics in Singapore entered a new phase in 2010 when Shen Zuowei of NUS was invited to give a 45-minute lecture at the ICM in Hyderabad, in the Numerical Analysis and Scientific Computing section. While an ICM invitation may not be considered a major event in a country with a long mathematical tradition, it nevertheless marked “the coming of age” for mathematics in Singapore, as the invitation was based on work done at NUS.

Shen Zuowei, ICM 2010⁸

The international recognition continued four years later at the ICM 2014 in Seoul, where four members from NUS (Bao Weizhu, Gan Wee Teck, Shen Weixiao and Yu Shih-Hsien) were invited to give 45-minute lectures (in the sections of Mathematics in Science and Technology, Number Theory, Dynamical Systems and Ordinary Differential Equations, and Partial Differential Equations, respectively). In the development of mathematics in the country, this was a high point and an historic achievement.

Indeed, the progress was dramatic overall. By 2015, members of the two departments of mathematics were collectively on no less than 60 editorial boards of major mathematical journals. Publishing in the most elite pure mathematics journals such as the *Annals of Mathematics*, the *Journal of the American Mathematical Society*, and



Gan Wee Teck, ICM 2014⁵



Bao Weizhu, ICM 2014⁹

Inventiones Mathematicae was no longer considered an unattainable goal, as demonstrated by several

colleagues. Invitations to speak at major conferences in one's research area were no longer a novelty. Several members have held senior positions in international scientific societies. For example, Louis Chen served as President of the Institute of Mathematical Statistics as well as President of the Bernoulli Society, while Chong Chi Tat was a member of the Executive Committee of the Council of the Association for Symbolic Logic. A few have their names affixed to mathematical terms, e.g. the Chen-Stein method in probability, the Ron-Shen duality principle in time-frequency analysis, and the Gan-Gross-Prasad conjecture in representation theory and number theory.

⁸ Photo courtesy of Sherman Riemenschneider

⁹ Photo courtesy of Bao Weizhu

Along with advances in pure mathematics, research in applied and computational mathematics at the NUS went through a major transformation during the last 25 years. In many respects this was in line with the rapid changes one witnessed in the role mathematics—particularly applied and computational mathematics—played in modern science and technology. From the early 1990's, the Department sought to strengthen its group in these areas. Today it is known internationally for its research in imaging science, scientific computing, operations research and quantitative finance. Members of the group serve on the editorial boards of top journals in the fields, and Shen Zuowei was one of the 27 invited speakers at the International Congress of Industrial and Applied Mathematics (ICIAM) in 2015. The ICIAM, held once every four years like the ICM, is the most prestigious international meeting in applied mathematics.



Shen Weixiao and Yu Shih-Hsien⁵

The first 50 years of mathematics in Singapore since independence ended on a high note. Looking back, this came as the combined effort of many individuals with the shared vision that Singapore would one day make a presence on the world mathematical stage. The journey was arduous but not without its rewards. The next 50 years will be even more challenging: in a world of steep competition for talent and excellence, maintaining the current position will be hard and making progress will be harder still. There is no assurance that the path taken will still lead to trees that bear fruits. And a structure that was painstakingly erected may still crumble over time. How history will record the first 100 years of mathematics in post-independence Singapore will depend on how the journey will be taken, and how the department's course is charted, in the coming decades.

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