RESEARCH BREAKTHROUGHS
DEPARTMENT OF MATHEMATICS

Rigidity Behind The Chaos

RESEARCHER
Associate Professor Zhang De-Qi

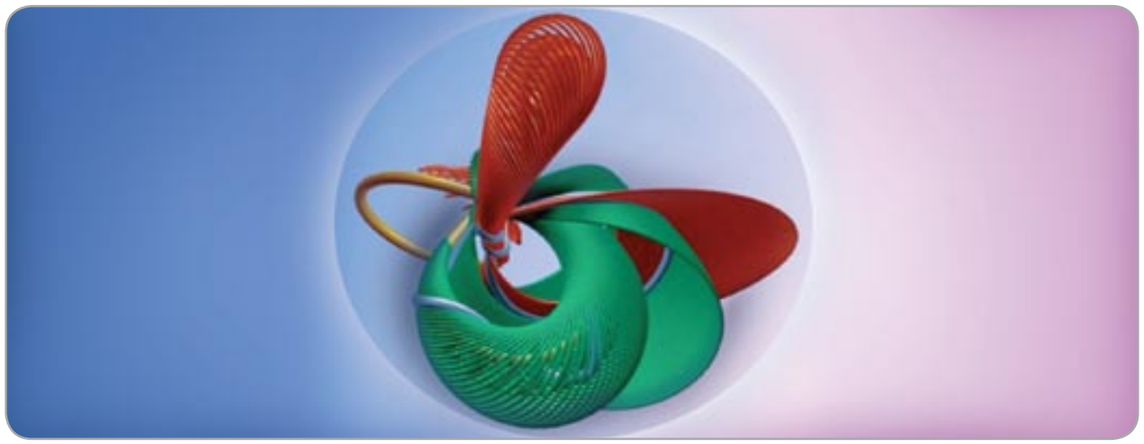

FIGURE 1 | The 3D-picture above shows the orbit of two intersecting curve segments (green and red) under a dynamically interesting matrix action.

(SOURCE: http://www.ams.org/featurecolumn/archive/lorenz.html)

Chaos is a field in mathematics that involves the study of dynamical systems, i.e. systems whose states evolve with time. The true nature underlying the chaotic phenomena can be elucidated by the study of dynamics. This will allow one to achieve control of the phenomena, for example, controlling the dynamical airflow around a flying aircraft.

Associate Professor Zhang De-Qi's work in dynamics aims to determine the dynamical structure of symmetries on spaces or complex manifolds (mathematical spaces). He uses the deep link between chaos and rigidity to provide a geometric understanding of universal constants/constraints (i.e. topological entropy / geometric structure) in dynamics.

Zhang and his collaborator have successfully identified the basic building blocks (and the way to canonically rebuild all symmetries from them) of dynamics of symmetries on compact complex spaces. The results they obtained were foundational and this was the first time mathematicians were presented with a clear description of geometric structures confirming the existence of many symmetries on them.

He dealt with the dynamics of symmetries on manifolds **M** with the result that every compact Kähler manifold **M** of dimension **n** either has very chaotic symmetries, or it has at most **n-1** of dynamically significant independent symmetries (i.e. of positive topological entropy) commutative to each other. The result was a solution to the Dynamic Tits Conjecture which was difficult and had resisted previous attempts at solving it. Zhang arrived upon the solution by devising a slick and simple sequence of quasi-positive real currents.

Zhang's results were published in a series of 8 papers (2 written jointly with N. Nakayama) listed in http://arxiv.org/find/math/1/au:+Zhang_D/0/1/0/all/0/1. Among the 8 papers, one titled "A Theorem of Tits type for Compact Kähler Manifolds" carrying his solution to the Dynamic Tits Conjecture was published in ***Inventiones Mathematicae***, one of the top mathematics journals. He has been invited to discuss the results in this paper at the following conferences:

- **Hong Kong Geometry Colloquium**, jointly organised by 3 major Universities in Hong Kong
- **Dynamics & Complex Geometry** held at the Centre Internationale des Rencontres Mathématiques and jointly organised by the University of Paris 6 and 11
- Inaugural **Pacific Rim Mathematical Association (PRIMA) Congress**, Sydney

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INTERVIEW with Associate Professor Zhang De-Qi

Associate Professor Zhang De-Qi received his Bachelor of Science from East China Normal University (Shanghai), and subsequently earned his Master of Science and Ph.D. from Osaka University, Japan. Prior to joining NUS, he was a faculty member at Osaka University, holding an appointment which is equivalent to that of an Assistant Professor.

RESEARCHER | Assoc Prof Zhang De-Qi

Interviewer: Tell us a little bit about yourself, particularly regarding your choice to become a mathematician.

Associate Professor Zhang De-Qi: To tell the truth, probability of me becoming a mathematician was 50-50. I did relatively well in both chemistry and mathematics at high school. However, I eventually opted for mathematics and I am glad to say that it has been a wonderful choice. Things are much easier being a mathematician since everything has a clear answer, unless, of course, you study Fuzzy theory.

Which mathematician do you look up to as a role model? Why?

The late Chern Shiing-Shen, a great geometer who was also founder of the Berkeley Mathematical Sciences Research Institute, and discoverer of the Chern classes commonly used in all branches of mathematics. He kept a low profile and pursued fundamentally important studies, remaining mathematically active till his last days.

You and your collaborator successfully identified the basic building blocks of dynamics of symmetries on compact complex spaces. Your result is also the solution to the Dynamic Tits Conjecture. What was the process leading to the identification of the basic building blocks and Conjecture solution?

Around the end of the year 2005, Curtis McMullen (Fields medallist at Harvard University) informed me of his new paper where he had constructed a series of dynamically interesting (i.e. expanding in one direction while contracting in another direction) symmetries g on rational surfaces S (like some compact spaces containing the complex-two-variable plane). I managed to compute the full symmetry group of the surface S and found that basically g (and its powers) is the only dynamically interesting symmetry of the surface S. Subsequently, two thoughts struck me: why are there so few symmetries on the surface S, and why is it so rare for us to find dynamically interesting symmetries? The results of these queries were written down in my papers in the middle of the year 2006: one on the Dynamic Tits Conjecture for two dimensional spaces, and another on building blocks for invertible symmetries. Later on, together with N. Nakayama, the building blocks result was further extended to the case of non-invertible symmetries.

What was your reaction when you first learnt that your paper was accepted for publication in ***Inventiones Mathematicae***, a top mathematical journal?

I was delighted to receive the acceptance letter as it is not easy for a potential paper to be accepted by such a journal.

What advice do you have for students who wish to become mathematicians?

There is an old saying that mathematics is not about speed but rather about depth. Becoming a mathematician is not a matter of months or even years but rather a journey which requires cumulative and dedicated effort. Hope you enjoy your experience with mathematics!