Deblurring the unknown

Innovative mathematical techniques can restore missing information in photographs, leading to clearer images

Introduction

One challenging problem often seen in imaging sciences is the so-called blind deconvolution problem, which aims at recovering the clear image from one blurred observation without knowing the blurring process. Built upon the concept of sparse modelling/approximation, several new mathematical frameworks and computational techniques that are developed provide a strong theoretical foundation, as well as effective computational methods, for resolving the blind deconvolution problem. Moreover, these techniques can also be used for solving many other inverse problems in imaging sciences.

Despite the astonishing progress in digital imaging technology, the demand of higher image quality at lower hardware cost is still growing rapidly in many imaging applications. Image recovery aims at restoring images which are of better quality from input images in which important information might either be severely attenuated or are completely missing. Image recovery usually is an ill-posed inverse problem such that a direct reversing process cannot recover missing information and will amplify noise such that the resulting images contains many artifacts. In recent years, the author and his collaborators together with researchers from the Centre for Wavelets, Approximation and Information Processing (CWAIP) have been working on developing new mathematical frameworks and computational methods to provide deeper understanding and effective computational methods for solving real image recovery problems in practice. CWAIP is a research centre within the Faculty of Science to push forward the frontier of the areas of visual information processing, analysis, understanding, and apply state-of-the-art mathematical theory and computational technology to solve real-life problems (see page 12). In particular, the so-called blind deconvolution problem has been one main problem that the team is working on.

Image Blurring

Image blurring is one primary cause of low-quality images. Image blurring can be modelled as a convolution process which will eliminate or severally attenuate high-frequency component of images. Image de-blurring is about recovering the missing information and restoring attenuated information. De-blurring is an ill-posed inverse problem as a direct reversing process cannot recover the missing information and will amplify image noise such that the recovered image is severely distorted. Even worse, in many imaging applications, how input images are blurred is not known or cannot be accurately estimated by hardware. Thus, the problem of recovering a clear image from a blurred image is called a blind deconvolution problem, which is often seen in digital photography and scientific imaging.

Blind Image Deconvolution

Blind image deconvolution is a challenging problem with many solutions that are sound in Mathematics. One obnoxious solution is having a blurred image which is generated by itself and a trivial convolution process that does nothing. A fundamental question is how to define sharp images and how to estimate them from blurred ones. The author and his collaborators have developed several new mathematical tools and computational frames for resolving this challenging problem. The basic idea is to model sharp images and the kernels that determine the blurring process by sparse approximation, which assumes that the data of interest are compressible in some suitable transform domain. For a sparse approximation based blind image deconvolution system, there are three main components:

1) what transform can sparsely model data for estimation;

2) how to find the sparse representation of data under the given transform; and

3) how to solve the problem in a robust manner that takes all kinds of practical error sources into account.

Wavelet Tight Frames

Built upon the concepts of wavelet tight frames and sparse approximation via ℓ_1-norm relating optimisation, the author and his collaborators developed in [1] a powerful sparse approximation based framework for blind image deblurring. The proposed method alternatingly estimates the blur kernel and the sharp image via solving an ℓ_1-norm relating optimisation problem. Wavelet tight frame is used for sparsely approximating the image, as well as the blur kernel. In a subsequent work, it is observed that the ℓ_1-norm based sparse approximation is biased toward a slightly blurred result, and thus the ratio ℓ_1/ℓ_2 is proposed as the normalised sparsity prompting function for correcting the bias of the result using ℓ_1-norm based sparsity-prompting function. The proposed computational framework can effectively restore a large class of blurred images taken by hand-held cameras. Some of these techniques have drawn attention from the industry and some companies have assigned consultation contracts for the implementation of these published algorithms in their camera systems to improve image quality.

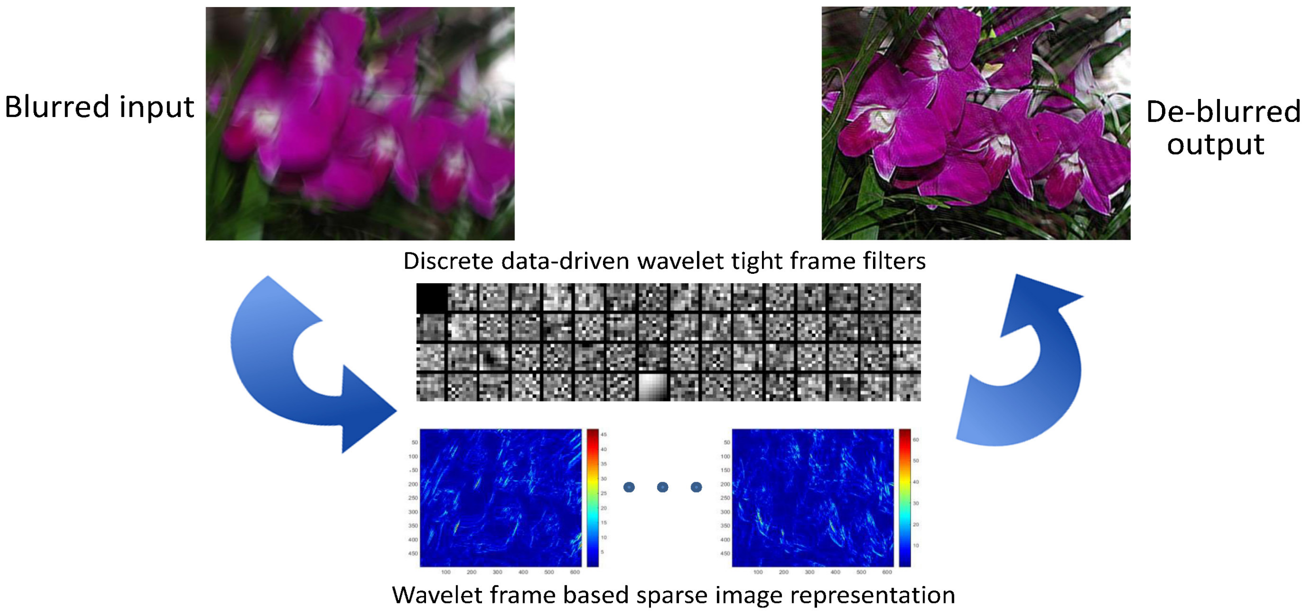

Figure 1: Blind image deblurring via sparse approximation.

Image Deconvolution

The convolution process for modelling image blurring is often a simplified model which does not exactly reflect the physical process happening in practice. A high-performance non-blind or blind deconvolution system that works well in practice needs to take such a model error into consideration. Built upon a new sparse approximation based variational model, a robust non-blind image de-convolution technique is developed in [2], the first available technique that is robust to model error. The basic idea is to simultaneously estimate three components:

1) sharp image;

2) the distortion of image gradients; and

3) the artifacts caused by kernel error.

All three components can be sparsely approximated under different transforms. The proposed method is applied on the non-stationary blind image deconvolution problem and it has showed superior performance over the existing techniques.

Data-driven Wavelet Tight Frames

One key for further improving the performance of state-of-the-art sparse approximation based image recovery method is to design new transforms that can be more efficient on sparsely approximating input images. Built upon the concept of "adaptive learning", a new approach is developed in [3] which constructs a class of data-driven wavelet tight frames that are optimised for the input data, instead of the existing generic wavelet tight frames. The data-driven wavelet tight frames constructed from the proposed approach can effectively capture the structures of input image data, which leads to better performance of sparse approximation to input image. Such an improvement over existing generic wavelet tight frames leads to noticeable performance gain in many wavelet based image recovery methods.

The techniques developed for blind image deconvolution and data-driven wavelet tight frame not only can be used for solving the related image recovery problems, but also can be applied to solve other inverse problems arising from imaging sciences, including 3D reconstruction in electronic microscopy and computed tomography, signal processing for high-dimensional data and many others.

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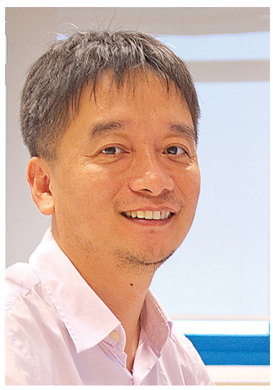

References

[1] J. Cai, H. Ji, C. Liu and Z. Shen, "Framelet based blind image de-blurring from a single image", *IEEE Transactions on Image Processing*, 21(2), 2012.

[2] H. Ji and K. Wang, "Robust image de-convolution with an inaccurate blur kernel", *IEEE Transactions on Image Processing*, 21(4), 2012.

[3] J. Cai, H. Ji, Z. Shen and G. Ye, "Data-driven tight frame construction and image de-noising", *Applied and Computational Harmonic Analysis*, 37(1), 89-105, Jul. 2014.