Math Masterclass

Why should students study linear algebra?

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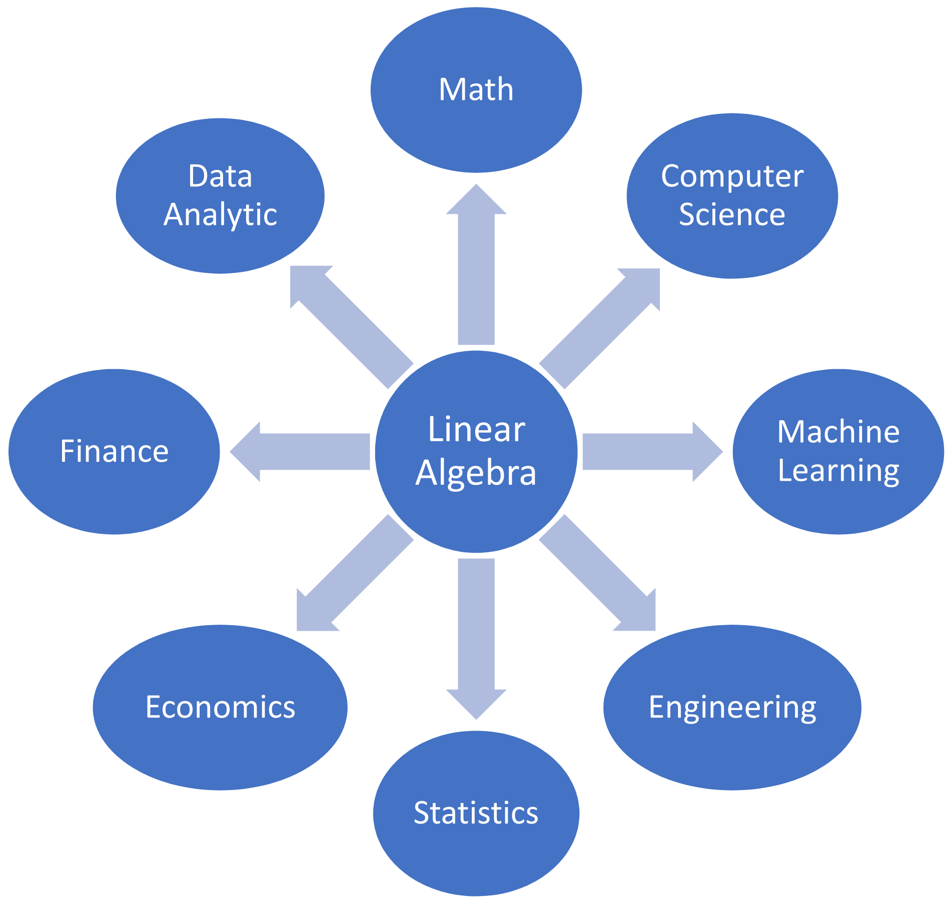
Linear Algebra is Everywhere



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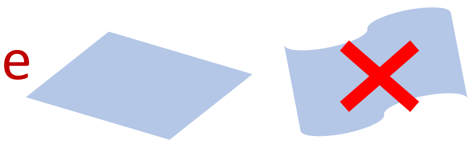
What is Linear Algebra?

having the properties of lines

1-dimensional linear object: line

2-dimensional linear object: plane



The study of mathematical objects and properties that are related to lines, planes and their generalization.

linear systems
matrices
vectors

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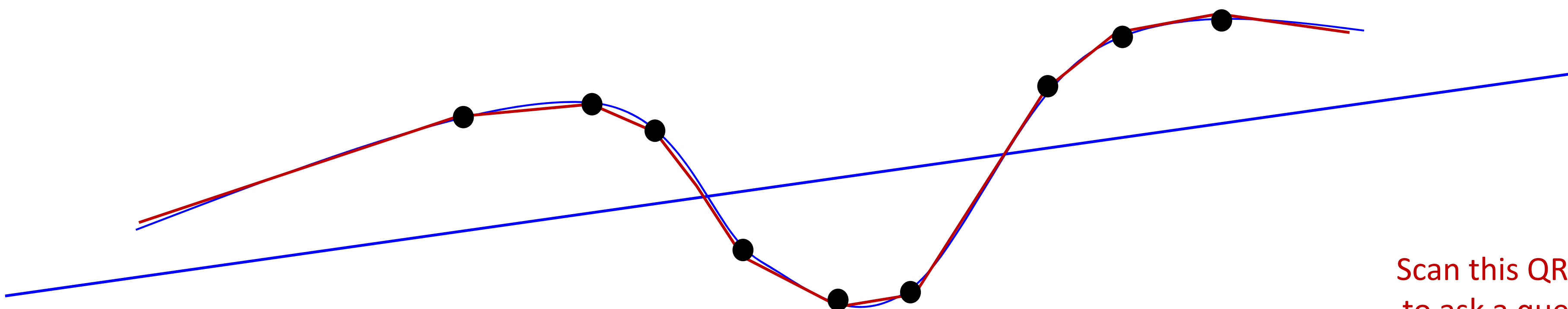
Why Linear?



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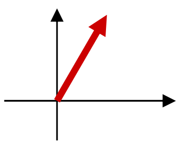
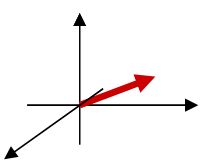
Vectors and Matrices





Dimension	Vector	Matrix
2-dimension		
Algebraic	$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}$
Geometric	arrow in xy-plane	transformation of a vector to another in xy-plane
3-dimension		
Algebraic	$\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 6 \\ 5 & 3 & 0 \\ 4 & 7 & 1 \end{pmatrix}$
Geometric	arrow in xyz-space	transformation of a vector to another in xyz-space
> 3-dimension		
Algebraic	$\begin{pmatrix} 2 \\ 4 \\ 1 \\ 7 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 6 & 4 \\ 5 & 3 & 0 & 8 \\ 4 & 7 & 1 & 2 \\ 0 & 0 & 9 & 1 \end{pmatrix}$
Geometric	None	None

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Linear Systems

Linear system
2 equations
2 variables

$$\begin{array}{rcrcr} x &+& y &=& 1 \\ 2x &+& 3y &=& 2 \end{array}$$

represented by 2 lines

solution: x = 1, y = 0

intersection of the 2 lines

algebraic generalization

Linear system
5 equations
4 variables

$$\begin{array}{rcrcrcrcr} x &+& y &+& z &+& w &=& 1 \\ 2x &-& y &+& 3z &+& 5w &=& 2 \\ x &+& 2y &+& 7z &+& 0w &=& 5 \\ 0x &-& 6y &+& 2z &+& 9w &=& 0 \\ 5x &+& 2y &-& 4z &+& 7w &=& 8 \end{array}$$

How to find the solutions?

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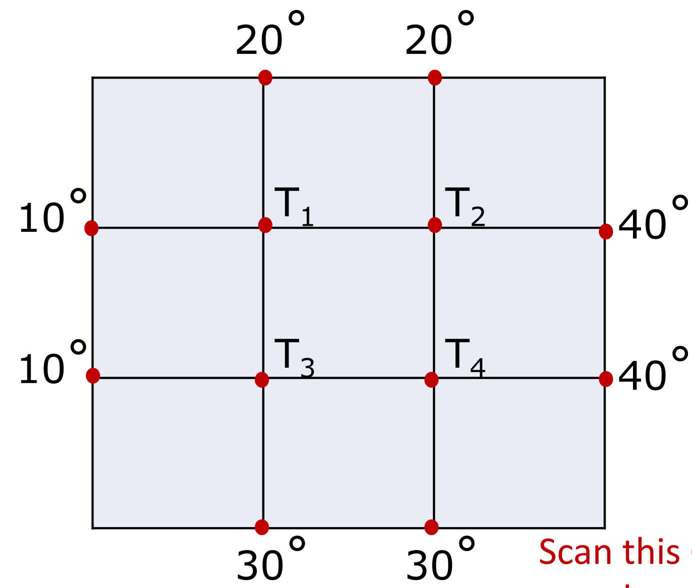
Heat Transfer

Steady-State temperature distribution of a thin plate

Temperature around boundary is known.

T_1, T_2, T_3, T_4 are temperatures at the 4 internal nodes.

Find T_1, T_2, T_3, T_4.



Known fact: Temperature at a node is approximately equal to the average of the 4 nearest nodes (on the left, right, above, below).

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Heat Transfer

Set up a Linear System in variables T_1, T_2, T_3, T_4.

Node 1: $T_1 = (10 + 20 + T_2 + T_3)/4$

$4T_1 - T_2 - T_3 = 30$

Node 2: $T_2 = (20 + 40 + T_4 + T_1)/4$

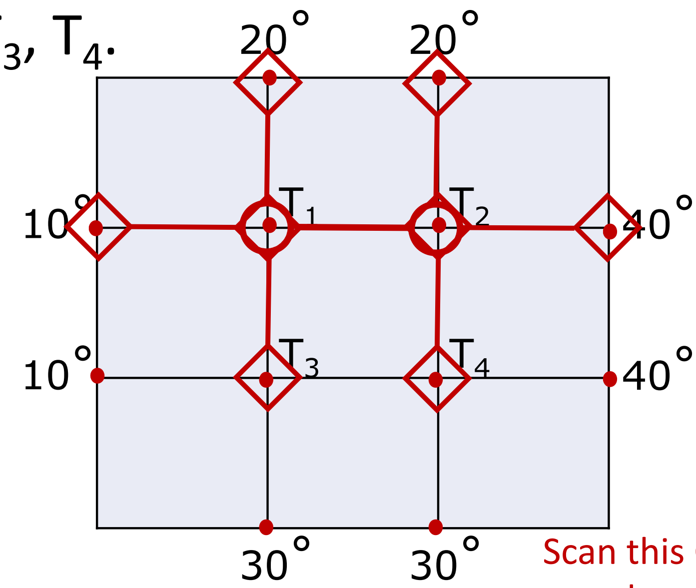
$-T_1 + 4T_2 - T_4 = 60$

Node 3: $T_3 = (10 + 30 + T_4 + T_1)/4$

$-T_1 + 4T_3 - T_4 = 40$

Node 4: $T_4 = (30 + 40 + T_2 + T_3)/4$

$-T_2 - T_3 + 4T_4 = 70$



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Matrix Equation

$$\begin{array}{ccccccccc} x & + & y & + & z & + & w & = & 1 \\ 2x & - & y & + & 3z & + & 5w & = & 2 \\ x & + & 2y & + & 7z & + & 0w & = & 5 \\ 0x & - & 6y & + & 2z & + & 9w & = & 0 \\ 5x & + & 2y & - & 4z & + & 7w & = & 8 \end{array}$$

standard form

$\Longleftrightarrow$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 3 & 5 \\ 1 & 2 & 7 & 0 \\ 0 & -6 & 2 & 9 \\ 5 & 2 & -4 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 5 \\ 0 \\ 8 \end{pmatrix}$$

matrix equation form

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Manufacturing

Three products A, B, C

Production cost per item

	A	B	C
Raw materials	0.10	0.30	0.15
Labor	0.30	0.40	0.25
Overhead & Misc.	0.10	0.20	0.13

Production cost matrix

$$\begin{pmatrix} 0.10 & 0.30 & 0.15 \\ 0.30 & 0.40 & 0.25 \\ 0.10 & 0.20 & 0.13 \end{pmatrix}$$

Easier to manipulate the data

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Manufacturing

Production cost per item

	A	B	C
Raw materials	0.10	0.30	0.15
Labor	0.30	0.40	0.25
Overhead & Misc.	0.10	0.20	0.13

produce: 3000 of A
2000 of B
5000 of C

$$\begin{pmatrix} 0.10 & 0.30 & 0.15 \\ 0.30 & 0.40 & 0.25 \\ 0.10 & 0.20 & 0.13 \end{pmatrix} \begin{pmatrix} 3000 \\ 2000 \\ 5000 \end{pmatrix} = \begin{pmatrix} 1650 \\ 2950 \\ 1350 \end{pmatrix}$$

total raw material cost
total labor cost
total overhead cost

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Manufacturing

Production cost per item

	A	B	C
Raw materials	0.10	0.30	0.15
Labor	0.30	0.40	0.25
Overhead & Misc.	0.10	0.20	0.13

Given budget:
2000 for raw material
2500 for labor
1500 for overhead cost

How many of A, B, C can be produce?

$$\begin{pmatrix} 0.10 & 0.30 & 0.15 \\ 0.30 & 0.40 & 0.25 \\ 0.10 & 0.20 & 0.13 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2000 \\ 2500 \\ 1500 \end{pmatrix}$$

Linear system in matrix equation form

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0.10 & 0.30 & 0.15 \\ 0.30 & 0.40 & 0.25 \\ 0.10 & 0.20 & 0.13 \end{pmatrix}^{-1} \begin{pmatrix} 2000 \\ 2500 \\ 1500 \end{pmatrix}$$

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Population Model

In a certain town, 30% of married women get divorced each year and 20% of women get married each year.
There are 8000 married women and 2000 single women.
Assuming that the total population of women remains constant, how many married women and how many single women will there be after n years?

$M_n =$ married women after n years

$S_n =$ single women after n years

$$M_n = 0.7M_{n-1} + 0.2S_{n-1}$$
$$S_n = 0.3M_{n-1} + 0.8S_{n-1}$$

Populations between two consecutive years

$$\begin{pmatrix} M_n \\ S_n \end{pmatrix} = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix} \begin{pmatrix} M_{n-1} \\ S_{n-1} \end{pmatrix}$$

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Population Model

In a certain town, 30% of married women get divorced each year and 20% of women get married each year.
There are 8000 married women and 2000 single women.
Assuming that the total population of women remains constant, how many married women and how many single women will there be after n years?

After 1 year

$$\begin{pmatrix} M_1 \\ S_1 \end{pmatrix} = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix} \begin{pmatrix} 8000 \\ 2000 \end{pmatrix} = \begin{pmatrix} 6000 \\ 4000 \end{pmatrix}$$

After 2 year

$$\begin{pmatrix} M_2 \\ S_2 \end{pmatrix} = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix} \begin{pmatrix} 6000 \\ 4000 \end{pmatrix} = \begin{pmatrix} 5000 \\ 5000 \end{pmatrix}$$

$$= \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}^2 \begin{pmatrix} 8000 \\ 2000 \end{pmatrix}$$

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Population Model

In a certain town, 30% of married women get divorced each year and 20% of women get married each year.
There are 8000 married women and 2000 single women.
Assuming that the total population of women remains constant, how many married women and how many single women will there be after n years?

After n year

$$\begin{pmatrix} M_n \\ S_n \end{pmatrix} = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix} \cdots \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix} \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix} \begin{pmatrix} 8000 \\ 2000 \end{pmatrix}$$

$$= \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}^n \begin{pmatrix} 8000 \\ 2000 \end{pmatrix}$$

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Google Page Rank

Google ranks webpages according to “hyperlinks”

e.g. we want to rank 4 webpages: A, B, C, D

Form a 4x4 matrix:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 1 & \frac{1}{3} & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & \frac{1}{2} & 0 \end{pmatrix} \end{array} \xrightarrow{\text{eigenvector}} \begin{pmatrix} 0.446 \\ 0.223 \\ 0.743 \\ 0.446 \end{pmatrix}$$

Eigenvector	page rank
0.446	2 (tie)
0.223	4
0.743	1
0.446	2 (tie)

A has a link to C, but not to B and D

B has a link to A, C, D

C has a link to A, D

D has a link to B, C

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