NUS National University of Singapore | Department of Mathematics Faculty of Science

WORKSHOP

REPRESENTATION THEORY, COMBINATORICS AND DEEP LEARNING

Organizer: Huanchen Bao (National University of Singapore)

THURSDAY 20 OCTOBER 2022 | NUS LT 32

PROGRAMME

0900 - 1000	*Anna Romanov (UNSW Sydney)* A Soergel bimodule approach to the character theory of real groups
1010 - 1110	*Jinfeng Song (National University of Singapore)* Quantum Frobenius Splittings for Quantum Symmetric Pairs
1120 - 1220	*Chaim Even-Zohar (Technion)* The BCFW Triangulation of the Amplituhedron
1220 - 1430	**Lunch Break**
1430 - 1530	*Xuhua He (Chinese University of Hong Kong)* The G-variations
1530 - 1600	**Coffee Break**
1600 - 1700	*Geordie Williamson (University of Sydney)* What can the working mathematician expect from deep learning?

ABSTRACTS

ANNA ROMANOV

Title: A Soergel bimodule approach to the character theory of real groups

Abstract: Admissible representations of real reductive groups are a key player in the world of unitary representation theory. The characters of irreducible admissible representations were described by Lusztig—Vogan in the 80's in terms of a geometrically-defined module over the associated Hecke algebra. In this talk, I'll describe a categorification of a block of the LV module using Soergel bimodules. This is joint work with Scott Larson.

JINFENG SONG

Title: Quantum Frobenius Splittings for Quantum Symmetric Pairs

Abstract: There is a map from the classical universial enveloping algebra to the quantum group at roots of unity, which splits the quantum Frobenius homomorphism (quantum Frobenius splitting). This map gives rise to a splitting of the p-th power map (Frobenius splitting) for the coordinate ring of the reductive group over algebraically closed field of positive characteristic. It also induces a Frobenius splitting of the flag variety, which compatibly splits Schubert divisors.

We generalise the construction of the quantum Frobenius splittings to quantum symmetric pairs of quasi-split types. We show that this map induces a Frobenius splitting for the symmetric subgroup K, and a (new) Frobenius splitting for the flag variety, which compatibly splits K-stable divisors. Along the way, we will also introduce a group scheme over Z[2^{-1}], which generalises Lusztig's construction for the Chevalley groups using quantum groups. This is work in progress with Huanchen Bao.

CHAIM EVEN-ZOHAR

Title: The BCFW Triangulation of the Amplituhedron

Abstract: The amplituhedron A(n,k,m) is a geometric object, discovered by Arkani-Hamed and Trnka (2013) in the study of scattering amplitudes in quantum field theories. They conjectured that A(n,k,4) admits a decomposition based on a certain combinatorial structure. The components are images of BCFW positroid cells of the Grassmannian Gr(k,n), which arise from the Britto–Cachazo–Feng–Witten recurrence (2005). In a recent paper with Tsviqa Lakrec and Ran Tessler, we prove this conjecture. In the talk, I will review the amplituhedron, its BCFW triangulation, and some ideas from the proof.

ABSTRACTS

XUHUA HE

Title: The G-variations

Abstract: The flag varieties (of a reductive group) play a crucial role in geometry and representation theory. The flag varieties have decompositions into the Schubert cells and the opposite Schubert cells. Moreover, Lusztig introduced a positive structure on the flag varieties, which is compatible with the decompositions above. The totally nonnegative flag varieties have many nice geometric and representation-theoretic properties, and Lusztig referred to them as ``remarkable polyhedral spaces". Recently, Galashin, Karp, and Lam proved that the totally nonnegative flag varieties are regular CW complexes homeomorphic to closed balls.

There are many other interesting varieties related to reductive and (more generally) Kac-Moody groups, which admit nice decompositions and positive structures. One wonders if they are also ``remarkable polyhedral spaces". In this talk, I will explain the atlas map to relate some of these varieties with the flag variety of a (large) Kac-Moody group and how to use the atlas map to establish some nice properties of these varieties. This talk is based on recent and ongoing joint works with Huanchen Bao.

GEORDIE WILLIAMSON

Title: What can the working mathematician expect from deep learning?

Abstract: Deep learning (the training of deep neural nets) is a simple idea, which has had many extraordinary applications throughout industry and science over the last decade. In mathematics the impact has so-far been modest at best. I will discuss a few instances where it has proved useful, and led to interesting (pure) mathematics. I will also discuss what can be learned from these examples, and try to guess an answer to the question in the title. I will also reflect on my experience as a pure mathematician interacting with deep learning. (Deep learning also raises interesting mathematical questions, but this talk won't be about this.)