Department of Mathematics
Faculty of Science

Research Highlight: Sparse Multi-Reference Alignment : Phase Retrieval, Uniform Uncertainty Principles and the Beltway Problem

Author: Subhroshekhar Ghosh

Motivated by cutting-edge applications like cryo-electron microscopy (cryo-EM), the Multi-Reference Alignment (MRA) model entails the learning of an unknown signal from repeated measurements of its images under the latent action of a group of isometries and additive noise of magnitude σ. Despite significant interest, a clear picture for understanding rates of estimation in this model has emerged only recently, particularly in the high-noise regime σ ≫ 1 that is highly relevant in applications.

Recent investigations have revealed a remarkable asymptotic sample complexity of order σ6 for certain signals whose Fourier transforms have full support, in stark contrast to the traditional σ2 that
arise in regular models. Often prohibitively large in practice, these results have prompted the investigation of variations around the MRA model where better sample complexity may be achieved.

In this paper, we show that sparse signals exhibit an intermediate σ4 sample complexity even in the classical MRA model. Further, we characterise the dependence of the estimation rate on the support size s as Op(1) and Op(s3.5) in the dilute and moderate regimes of sparsity respectively. Our techniques have implications for the problem of crystallographic phase retrieval, indicating a certain local uniqueness for the recovery of sparse signals from their power spectrum. Our results explore and exploit connections of the MRA estimation problem with two classical topics in applied mathematics: the beltway problem from combinatorial optimization, and uniform uncertainty principles from harmonic analysis. Our techniques include a certain enhanced form of the probabilistic method, which might be of general interest in its own right.

Keywords: Multi reference alignment · Cryo electron microscopy · Phase retrieval · Uncertainty principles · Fourier analysis · Beltway problem · Combinatorial optimization · Sparse signal processing · Sample complexity · Probabilistic method

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