9TH

NUS GRADUATE SYMPOSIUM IN MATHEMATICS

13 May 2025 (Tuesday)
Department of Mathematics
S17 #04-05 & #04-06

PROGRAMME

Time/Venue	S17-04-06	
08:50 – 09:00	**Opening Address** Prof Gan Wee Teck	
09:00 – 09:45	**Plenary I: Diffusion Models for High Dimensional Distributions** A/Prof Xin T. Tong *p2*	
09:45 – 10:15	*Refreshment break @ Mathematics Department Lounge*	
10:15 – 11:00	**Plenary II: Patterson-Sullivan measures for transverse groups** A/Prof Zhang Tengren *p2*	
Time/Venue	S17-04-06 (Pure Math)	S17-04-05 (Applied Math)
11:00 – 11:25	Xue Jingyi *p2*	Elahe Khayatian *p2*
11:25 – 11:50	Song Jinfeng *p3*	Hou Di *p3*
12:00 – 13:45	*Lunch @ Mathematics Department Lounge*	
13:45 – 14:10	He Taoran *p4*	Yunlong Hou *p3*
14:10 – 14:35	Li Yuanchu *p4*	Zhao Jiaxi *p4*
14:40 – 15:00	Peng Fei *p5*	Shuigen Liu *p5*
15:00 – 15:30	*Refreshment break @ Mathematics Department Lounge*	
15:30 – 15:55	Loh Jia Sheng, Colin *p5*	Li Linfeng *p5*
15:55 – 16:20	Mingjie Wang *p6*	Hannah L. H. Lai *p6*
16:20 – 16:45		Tan Kin Aun *p6*

(Numbers in italic denote the page number of abstract)

ABSTRACTS

A/Prof Xin T. TONG

Diffusion Models for High Dimensional Distributions

Diffusion model is a popular tool to generate new data samples. However, rigorous understanding of diffusion model is still lacking. One issue is how to train these models for high dimensional problems as score function estimation is subject to the curse of dimension. Another issue is how to avoid the memorization effect, where the diffusion model is bound to generate an exact copy from the training data. We will provide solutions to the first issue by focusing on high dimensional distributions with sparse dependence. We will leverage the sparse dependence to provide a local estimation of the score functions. As for the second issue, we will modify the diffusion model in the final stage and generate new samples close to the same manifold where the training data originated.

A/Prof ZHANG Tengren

Patterson-Sullivan measures for transverse groups

I will recall the classical Patterson-Sullivan theory for hyperbolic surfaces and indicate how this can be generalized to transverse subgroups of higher rank Lie groups. This is joint work with Richard Canary and Andrew Zimmer.

XUE Jingyi

Flows on the SO(3,2)-Hitchin component

Let S be a closed surface with genus at least 2. Given any maximal geodesic lamination λ of S with finitely many leaves, Bonahon-Dreyer constructed a parameterization of the Hitchin component $Hit\big(S, PSL_n(\mathbb{R})\big)$. Later on, Sun-Wienhard-Zhang constructed new flows on the Hitchin component for $PGL(V)$. The flows they defined involve shearing flows that are associated to leaves of λ and eruption flows that are associated to pair of pants in S. Following their works, we first construct new shear invariants and triangle invariants for $SO(3,2)$-Hitchin representations. These parameters will behave well when we perform special shearing flows and eruption flows. Since $Sp(4,\mathbb{R})$ is a double cover of $SO(3,2)$, we also obtain invariants and flows for $PSp(4,\mathbb{R})$-Hitchin representations. This is a joint work with my supervisor Prof. Zhang Tengren.

Elahe KHAYATIAN

Simple K-RF Metrics for Comparison of Labeled DAGs

Causal relationships are frequently represented as labeled directed acyclic graphs (DAGs) across various scientific disciplines. In particular, such structures are used to model the progression of malignant tumor cells. A robust metric for comparing these structures is essential not only for evaluating different models but also for developing or enhancing DAG inference methods. However, defining an efficient metric in the space of labeled DAGs remains a significant challenge. In this work, we introduce novel dissimilarity measures to address this issue. These measures refine the k-Robinson-Foulds distances, originally developed for comparing labeled trees, and demonstrate improved efficiency in comparing DAGs with varying label sets compared to existing methods.

SONG Jinfeng

Cluster algebras and total positivity

Cluster algebras are commutative rings with a set of distinguished generators having a remarkable combinatorial structure. Since their introduction by Fomin and Zelevinsky in 2000, they have found deep applications in many fields of mathematics, from Lie theory, Poisson geometry to triangulations of surfaces and Teichmüller theory.

Totally positive matrices are those whose minors are all positive. They were introduced by applied mathematicians Schoenberg and Grantmacher-Krein in 1930s, and has been largely generalized by Lusztig in 1994. In this talk, I will give an introduction to cluster algebras and explain their connections to total positivity.

HOU Di

A Low-Rank ALM for Doubly Nonnegative Relaxations of Mixed-Binary QP

Doubly nonnegative (DNN) programming problems are known to be challenging to solve because of their huge number of $O(n^2)$ constraints and $O(n^2)$ variables. In this work, we introduce RiNNAL, a method for solving DNN relaxations of large-scale mixed-binary quadratic programs by leveraging their solutions' possible low-rank property. RiNNAL is a globally convergent Riemannian based augmented Lagrangian method (ALM) that penalizes the nonnegative and complementarity constraints while preserving all other constraints as an algebraic variety. After applying the low-rank decomposition to the ALM subproblem, its feasible region becomes an algebraic variety with favorable geometric properties. Our low-rank decomposition model is different from the standard Burer-Monteiro (BM) decomposition model in that we make the crucial step to equivalently reformulate most of the quadratic constraints after the BM decomposition into fewer and more manageable affine constraints. This modification is also important in helping us to alleviate the violation of Slater's condition for the primal DNN problem. Moreover, we make the crucial step to show that the metric projection onto the algebraic variety, although non-convex, can be transformed into a tractable convex optimization problem under certain regularity conditions. Numerous numerical experiments are conducted to validate the efficiency of the proposed RiNNAL method. [Joint work with Dr. Tianyun Tang and Prof. Kim-Chuan Toh]

Yunlong HOU

BanditSpec: Adaptive Speculative Decoding via Bandit Algorithms

Speculative decoding has emerged as a popular method to accelerate the inference of Large Language Models (LLMs) while retaining their superior text generation performance. Previous methods either adopt a fixed speculative decoding configuration regardless of the prefix tokens, or train draft models in an offline or online manner to align them with the context. This paper proposes a training-free online learning framework to adaptively choose the configuration of the hyperparameters for speculative decoding as text is being generated. We first formulate this hyperparameter selection problem as a Multi-Armed Bandit problem and provide a general speculative decoding framework BanditSpec. Furthermore, two bandit-based hyperparameter selection algorithms, UCBSpec and EXP3Spec, are designed and analyzed in terms of a novel quantity, the stopping time regret. We upper bound this regret under both stochastic and adversarial reward settings. By deriving an information-theoretic impossibility result, it is shown that the regret performance of UCBSpec is optimal up to universal constants. Finally, extensive empirical experiments with LLaMA3 and Qwen2 demonstrate that our algorithms are effective compared to existing methods, and the throughput is close to the oracle best hyperparameter in simulated real-life LLM serving scenarios with diverse input prompts.

HE Taoran

On Kerr black hole formation and a new approach toward Penrose inequality

The study of the formation of black hole in gravitational collapse presents one of the most profound and challenging problems in mathematical general relativity, involving the deep understanding of nonlinear wave equations, geometric analysis and mathematical physics. In this talk, I will present our recent result on the construction of Kerr black hole formation spacetimes and a new method for proving the Penrose inequality in perturbed Kerr regime. Specifically, we show that $3+1$ dimensional Einstein vacuum equations admit dynamical Kerr black hole formation solutions with no symmetry assumptions arising from a class of admissible characteristic initial data. Our hyperbolic arguments combine the scale-critical gravitational-collapse result by An--Luk with the recent breakthrough by Klainerman--Szeftel on proving nonlinear Kerr stability with small angular momentum, which requires performing various specific coordinate changes and frame transformations. Furthermore, allowing large spacetime angular momentum, with new elliptic arguments and precise leading order calculations, we also solve the apparent horizon in Kerr black hole formation spacetimes (including Klainerman--Szeftel's Kerr stability spacetimes) and conduct an exploration, detailing the emergence, evolution, asymptotics and final states of the apparent horizon. Building on our analysis, without time symmetric assumption, we then put forward a new mathematical framework and prove both the dynamical Penrose inequality and the spacetime Penrose inequality in our black-hole formation spacetimes and in the perturbative regime of sub-extremal Kerr black holes. Collectively, without assuming any symmetry assumptions, our results extend Christodoulou's celebrated trapped surface formation theorem to a version being global in time with black hole formation. This is joint work with Xinliang An.

LI Yuanchu

Differential (co)homology and Anderson Duality

We introduce the notion of differential refinement for a generalized (co)homology theory, and present some examples such as the ordinary cohomology, K-theory and Spin^c cobordism theory. After that, we sketch the geometric construction of Anderson duals following Yamashita's work.

ZHAO Jiaxi

Combating distribution shift in scientific machine learning

Data-driven methods have emerged as promising tools for the reduced-order modeling in various scientific applications, e.g. turbulence modeling in computational fluid dynamics and XC functionals for density functional theory. Typically, machine-learning models are usually trained offline from the high fidelity data, they are then combined with classical solvers for simulations in new scenarios where severe instabilities may arise. We provide a theoretical framework to explain the stability from a dynamical system perspective and propose two solutions. The first one takes into account the underlying data manifold structure to ensure stabilities while the second one leverages generative models to capture the uncertainty in the reduced-order modeling. Theoretical guarantees are proved to justify the effectiveness of these methods and numerical experiments over transient fluid simulations show the promising performance improvement.

PENG Fei

An improved bound on Seymour's second neighborhood conjecture

Seymour's celebrated second neighborhood conjecture, now more than thirty years old, states that in every oriented digraph, there is a vertex u such that the size of its second out-neighborhood $N^{++}(u)$ at least as large as that of its first out-neighborhood $N^{+}(u)$. In this paper, we prove the existence of u for which $|N^{++}(u)| \geq 0.715538|\mathrm{N}^{+}(u)|$. This result provides the first improvement to the best known constant factor in over two decades.

Shuigen LIU

Localization Method for High-Dimensional Distribution Approximation

Many spatial models exhibit locality structures that effectively reduce their intrinsic dimensionality, enabling efficient approximation and sampling of high-dimensional distributions. However, existing approximation techniques mainly focus on joint distributions, and do not guarantee accuracy for low-dimensional marginals. By leveraging the locality structures, we establish a dimension independent uniform error bound for the marginals of approximate distributions. Inspired by the Stein's method, we introduce a novel δ-locality condition that quantifies the locality in distributions, and link it to the structural assumptions such as the sparse graphical models. The theoretical guarantee motivates the localization of existing sampling methods, as we illustrate through the localized likelihood-informed subspace method and localized score matching. We show that by leveraging the locality structure, these methods greatly reduce the sample complexity and computational cost via localized and parallel implementations.

LOH Jia Sheng, Colin

Period integrals associated to dual of strongly tempered hyperspherical variety

Recent work of Mao, Wan and Zhang have provided a complete list of strongly tempered hyperspherical varieties and they proposed some new period integrals. In this paper, I will present 9 new period integrals associated to dual of strongly tempered distinguished polarised hyperspherical varieties and discuss the L-functions these integrals represent, as examples of the Relative Langlands Duality.

LI Linfeng

Arbitrages in Perpetual Contracts

Perpetual contracts have gained widespread adoption in cryptocurrency markets, offering delta-one exposure to underlying coins without a fixed expiration date. The perpetual contract price is anchored to the underlying index price through recurrent funding swaps between long and short positions. This funding rate incentivizes trades to narrow the gap between perpetual contracts and underlying index prices.

Existing studies (Angeris et al., 2022; He et al., 2023; Ackerer et al., 2023) have studied how to price perpetual contracts. However, these works oversimplify the funding swap mechanism, limiting the practical applicability of their results. This paper establishes rigorous no-arbitrage bounds for both linear and inverse perpetual contracts by incorporating funding specifications and entry transaction costs. We demonstrate that arbitrage opportunities can be systematically exploited through dynamic trading strategies when perpetual contract prices deviate beyond these bounds. Additionally, we conduct an empirical study using data from Binance to validate the efficiency of our theoretical bounds.

Mingjie WANG

Quasilinear ODE with Measure Data

We study the singular problem

$$(-w|u'|^{p-2}u')' + \sigma|u|^{p-2}u = \mu, \quad \text{on } I$$

where $(0,1) \subseteq I \subseteq [0,1]$, $w > 0$ a.e., $w \in L^1_{\text{loc}}(I)$, $w^{-1/p-1} \in L^1_{\text{loc}}(0,1)$ and μ is a signed Radon measure on I while σ is a (positive) Radon measure on I under the assumption that

$$|u|_{L^\infty(I)} \leq C\left(|u'|_{L^p_w(I)} + |u|_{L^p_\sigma(I)}\right)$$

for all $u \in \mathcal{W}_0(I)$ where $\mathcal{W}_0(I)$ is the closure of $\text{Lip}_c(I)$ in the weighted Sobolev space

$$\mathcal{W}^{p}_{w,\sigma}(I) = \left\{u \in L^p_\sigma(I) \cap L^1_{\text{loc}}(I) : u' \in L^p_w(I)\right\}.$$

We show that this assumption is indeed both necessary and sufficient for the existence and uniqueness of weak solutions to the problem.

Hannah L. H. LAI

Neural Tangent Kernel in Implied Volatility Forecasting: A Nonlinear Functional Autoregression Approach

Forecasting implied volatility across different levels of moneyness and maturity is crucial yet challenging due to the high dimensionality of the Implied Volatility Surface (IVS) and the nonlinearity that characterizes its temporal dependence. We adopt a Nonlinear Functional Autoregressive (NFAR) framework to a sequence of IVS and employ neural networks that admit a Neural Tangent Kernel (NTK) parametrization to capture nonlinear interactions between surfaces. We illustrate the theoretical and numerical advantages of the proposed functional NTK (fNTK) estimator and establish a link to functional kernel regression. Our empirical analysis includes over 6 million European call and put options from the S&P 500 Index, covering January 2009 to December 2021. The results confirm the superior forecasting accuracy of the fNTK across different time horizons. When applied to short delta-neutral straddle trading, the fNTK achieves a Sharpe ratio ranging from 1.30 to 1.83 on a weekly to monthly basis, translating to 90% to 675% relative improvement in mean returns compared to forecasts based on functional Random Walk model.

TAN Kin Aun

Filtering Signal through a Topological Lens: Point Process and Persistent Homology on Time-Frequency Plane

We present a novel framework for analyzing time-dependent signals using Topological Data Analysis (TDA)—a powerful mathematical tool that extracts meaningful patterns from complex data. By studying the time-frequency representation of a signal, particularly the zeros of its spectrogram, we can detect and reconstruct signal components even in noisy environments. Our approach leverages persistent homology, a key concept in TDA, to identify robust structures in the data. This method proves especially effective in low signal-to-noise ratio (SNR) scenarios, outperforming traditional techniques. The talk will introduce these ideas in an intuitive way, demonstrating how advanced mathematics can solve real-world signal processing challenges.