Faculty of Science
Department of Mathematics

MA4198 PROJECT PROPOSAL (PROJECT CUM SEMINAR GROUP)

SUPERVISOR'S INFO

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PROJECT ID: PS2520-10

TITLE

When decision is possible

BRIEF DESCRIPTION OF PROJECT

Can one write a computer program that always outputs the correct **True/False** answer to any mathematical question? At first sight this seems plausible—after all, mathematics is often viewed as a perfectly logical discipline in which every statement must be either true or false. However, a landmark result of **Gödel**, known as the *Incompleteness Theorem*, shows that even when we restrict attention to statements purely about the natural numbers, no such program can theoretically exist. A later breakthrough in the resolution of **Hilbert's Tenth Problem** revealed an even stronger limitation: there is **no algorithm** that can decide whether an arbitrary Diophantine equation (say, with 17 variables) has an integer solution.

Despite these negative results, there are surprising and important areas of mathematics in which **decision is possible**. For example, it is known that there exists a computer algorithm that decides the truth of statements in **elementary Euclidean geometry**. More generally, certain mathematical theories are known to be **decidable**, meaning that there exists an algorithm which, given any statement in the language of the theory, determines whether it is true or false. These positive results rely on ideas from **model theory**, a branch of mathematical logic concerned with the relationship between formal theories and the mathematical structures they describe.

The aim of this project is to explore the concept of **decidability** in mathematics through accessible examples that illustrate when and why decision procedures exist. Students will study key ideas in a guided way and apply them to demonstrate decision in specific concrete contexts.

EXPECTATION/S

- Read and understand selected mathematical texts related to decidability and examples of decidable/undecidable theories.
- Gain familiarity with basic concepts from mathematical logic (at an introductory level, with guidance).
- Participate actively in weekly group discussions and short presentations.

- Work independently on an individual topic within the general theme of the project.

- Produce a clear and well-structured written report (up to 30 pages) explaining the material and demonstrating understanding of key ideas.

- Present the main results of the chosen topic in an accessible manner to peers.

- Apply decision techniques to specific concrete mathematical situations, where appropriate.

- Develop skills in mathematical exposition rather than solving open research problems.

PREREQUISITE/S (at level 3000 or below, with at most one course at level 3000)

- **Level-2000 modules (algebra/combinatorics background):**

  - MA2101 Linear Algebra II
  - MA2202 / MA2202S Algebra I
  - MA2214 Combinatorics and Graphs I

- One among these two l**evel-3000 module (recommended, but not required):**

  - MA3201 Algebra II
  - MA3233 Combinatorics and Graphs II

READING REFERENCE/S

``An Invitation to Mathematical Logic'' by David Marker