Faculty of Science
Department of Mathematics

MA4198 PROJECT PROPOSAL (PROJECT CUM SEMINAR GROUP)

SUPERVISOR'S INFO

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PROJECT ID: PS2610-07

TITLE

Normal Numbers and Randomness

BRIEF DESCRIPTION OF PROJECT

The notion of normality was defined by Émile Borel more than 100 years ago. A real number is normal in a given integer base b if in its expansion in base b all digits have the same asymptotic frequency and furthermore, all blocks of digits of equal length have the same asymptotic frequency. For example, if a number is normal in base two, each of the digits '0' and '1' occur in the limit, half of the times; each of the blocks '00', '01', '10' and '11' occur one fourth of the times, and so on. A real number that is normal in every integer base is called absolutely normal, or just normal. Borel proved that "with probability 1" all real numbers are normal. However, until now, all normal numbers are constructed. It remains open whether the fundamental mathematical constants such as π, $\sqrt{2}$ and e are normal to some integer base. Borel conjectured that all irrational algebraic numbers are normal, which is also wide open.

The project will follow the survey paper of Becher and Carton, where five different formulations of normality are given, and their equivalence are shown. Four of the definitions are combinatorial, and one is, in terms of finite automata, related to the notion of algorithmic randomness.

Further topics can be pursued depending on the interests of the project group. For example, we could learn the definition of Lebesgue measure and prove the aforementioned Borel's Theorem, where the quoted "with probability 1" is replaced by "with respect to Lebesgue measure, almost". Or we could learn how to construct a specific normal number.

EXPECTATION/S

To get a passing grade, one must know how to prove the equivalence of at least one pair of the formulations. To get a top grade, one needs to choose a further topic, say from the references of the survey paper, and present it convincingly to the group.

PREREQUISITE/S (at level 3000 or below, with at most one course at level 3000)

MA3210 Mathematical Analysis II and MA2202 Algebra I

READING REFERENCE/S

V. Becher and O. Carton. Chapter "Normal numbers and Computer Science" In "Sequences, Groups, and Number Theory", Valérie Berthé and Michel Rigó editors. Trends in Mathematics Series, Birkhauser/Springer, pp. 233-269, 2018.